# SIS Mixer to HEMT Amplifier Optimum Coupling Network

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*Abstract* — **The coupling network between a superconductor-insulator-superconductor (SIS) mixer and a high-electron-mobility-transistor (HEMT) amplifier is investigated from the point of view of minimizing the overall noise temperature and also increasing the saturation level of the mixer. The effect of a negative output impedance of the mixer upon the amplifier noise is considered and an optimum negative source resistance is found. The amplifier noise at this optimum negative source resistance is shown to be related to the noise wave coming out of the amplifier input terminals.**

*Key words:* **SIS, HEMT, low-noise, negative resistance.**

## I. Introduction

Superconductor-insulator-superconductor (SIS) mixers have become the device of choice for cryogenic (4 K) low-noise applications in the frequency range of 70 to 250 GHz [1]. The mixer is usually followed by a cryogenic low-noise FET or HEMT amplifier, and receiver single-sideband noise temperatures of under 100 K at 115 GHz have been achieved [2], [3]. Theoretical minimum noise temperatures are an order of magnitude lower [1] and experimental laboratory results with narrow-band, low-frequency IF amplifiers have confirmed this [4].

The general configuration discussed in this note is shown in Fig. 1. The design of an IF amplifier driven by an SIS mixer requires special consideration for the following reasons:

1) The output resistance of an SIS mixer may be negative and the effect of this negative source impedance upon the IF amplifier noise should be understood.

2) The SIS mixer has a very limited output voltage and saturates at a very low input power level (of the order of nW's), which can be increased if the IF amplifier input resistance is made very low.

3) The internal noise in the SIS mixer is very low and the receiver noise is critically dependent upon realizing a low-noise contribution from the IF amplifier.

4) For stability reasons, an SIS mixer with a negative output resistance may require an IF amplifier input impedance which is low or high dependent upon the reactance versus frequency slope in the circuit.

An important point that is often not realized by developers of SIS mixers is that the source impedance $Z_{opt}$ which minimizes the IF amplifier noise can be chosen independently of the IF amplifier input impedance $Z_{in}$. It has been common practice to use IF amplifiers which have been designed to have $Z_{opt} = Z_{in} = 50\ \Omega$. This is not required; $Z_{in}$ can be made low for reason 2) and $Z_{opt}$ can be chosen to give best noise performance. An example of a feedback amplifier designed for $Z_{opt} = Z_{in} = 50$ is given in [14]; the feedback and input network could be changed to give, for example, $Z_{opt} = 200$ and $Z_{in} = 20$.

In this paper we will assume that a lossless coupling network exists between the FET or HEMT device and the mixer; the impedance transformation of this network will then be optimized so that the resulting $Z_{opt}$ at the mixer plane minimizes the IF noise contribution. If the mixer has an output impedance, $Z_{out}$,

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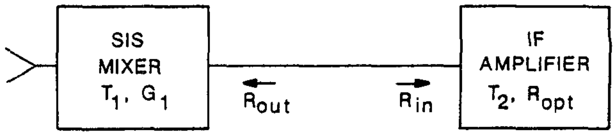


Fig 1. General configuration discussed in this note The mixer is described by noise temperature $T_1$, exchangeable gain $G_1$, and output resistance $R_{out}$. The amplifier is described by noise temperature $T_2$, which is a function of $R_{out}$; an input resistance $R_{in}$, which has no effect upon $G_1$ or noise performance; and an optimum generator resistance $R_{opt}$. Changing the impedance transformation of the amplifier input network changes both $R_{in}$ and $R_{opt}$, but by use of lossless feedback, $R_{in}$ can be changed independent of $R_{opt}$.

with positive real-part, we simply equate $Z_{opt} = Z_{out}$ and are finished. Lossless feedback would then be utilized to make the amplifier input impedance low; this feedback would have small effects upon $Z_{opt}$, but the input coupling network could be adjusted to compensate and preserve the $Z_{opt} = Z_{out}$ condition. We will find $Z_{opt}$ for the negative source resistance case and compute the resulting noise performance. Finally, the effects of transmission lines and isolators will be considered.

SIS mixers can be realized with positive output resistance, input match, and a small amount of gain (a few dB, SSB) [5]. The output resistance may be high, and an IF amplifier with high $R_{opt}$ and low $R_{in}$ appears to be desirable. If an isolator is inserted between the mixer and the IF amplifier, then $R_{opt} = R_{in}$ at the mixer-isolator interface [6]. Good results may be obtained, but the optimization which may result from $R_{opt} \neq R_{in}$ is not realized.

It is important to consider the effect of a transmission line of characteristic impedance $Z_0$ between the mixer and IF amplifier in the case $R_{opt} \neq R_{in}$. If $Z_0 = R_{opt} \neq R_{in}$, the noise performance will be independent of IF frequency, but the gain will depend upon the frequency-dependent transformation of $R_{in}$ caused by the transmission line. On the other hand, if $Z_0 = R_{in} \neq R_{opt}$, the gain will be independent of IF frequency, but the noise will not. The required solution is either to make the transmission line short (for wide bandwidth) and integrate the IF amplifier with the mixer, or to make the transmission line one-half wavelength long at the IF center frequency (with of the order of 30 percent bandwidth).

There may be no advantage to operating the SIS mixer with negative output resistance but should this occur, there are large effects which can increase or decrease the overall noise dependent upon the noise properties of IF amplifier active device; this topic is discussed in the next section.

## II. Negative Resistance Cascading Theory

The conventional noise figure cascading formula of Friis [7] uses concepts of available power and available gain. In the case of a negative output resistance, both of these quantities and the second-stage noise figure become infinite and the first-stage noise figure becomes indeterminate. Fortunately, this situation was recognized by Haus and Adler 30 years ago, and they made modifications to the theory [8], [9]. The modifications are somewhat strange and involve the concepts of exchangeable power and exchangeable gain, which are understandable mathematically but are somewhat obscure from a physical point of view. The important physical concept is that the conventional minimum noise temperature $T_{min}$ of an amplifier is the minimum with respect to variation of the source impedance *within the positive resistance plane*. When the source impedance is allowed to vary into the negative resistance plane, a new noise minimum noise temperature, which we will call $T_{neg}$, is revealed. In the formalism of Haus and Adler, $T_{neg}$ is also negative but is divided by a negative exchangeable gain to provide a positive contribution of the second stage to the overall noise temperature. For minimum noise, $|T_{neg}|$ should be minimized, and this is an entirely different case from the minimization of $T_{min}$. In terms of the correlated noise voltage- and current-source model of a noisy amplifier [10], the negative resistance allows complete cancellation of the correlated noise components from the two sources. In terms of the noise wave model of a noisy amplifier [11], the source reflection coefficient $>1$ produced by a negative resistance allows the noise wave coming out of the amplifier input to more completely cancel the correlated portion of the ingoing noise wave. The minimum noise temperature $T_{neg}$ does not occur when the cancelation is complete because the magnitude of the uncorrelated noise must also be considered.

A formal application of the Haus/Adler noise theory to the case of an SIS mixer is as follows. The exchangeable power $P_e$ of a current source $I_s$ with internal shunt resistance $R_s$ is given by

$$P_e = |I_s|^2 \cdot R_s/4 \tag{1}$$

and the exchangeable gain $G_1$ of the mixer is the ratio of exchangeable power at the IF output terminals to exchangeable power of the RF source. If $R_s$ is negative, the exchangeable power is negative; if $R_s$ is positive, the exchangeable power is positive and is equal to the available power of the source. For an SIS mixer with negative IF output resistance driven from a positive RF source resistance, the exchangeable gain is negative. In terms of the gain $G_0$ into normalizing impedance $Z_0$ (typically 50 Ω), $G_1 = G_0/(1-|\Gamma_{out}|^2)$, where $\Gamma_{out} = (Z_{out} - Z_0)\ (Z_{out} + Z_0)$ is the output reflection coefficient of the mixer.

The noise temperature of the mixer, $T_1$, without IF amplifier is given by

$$T_1 = N_e/(G_1 \cdot k \cdot \Delta f) \tag{2}$$

where $N_e$ is the exchangeable noise power at the mixer IF output terminals due to noise sources within the mixer.

The noise temperature of the cascade of mixer and IF amplifier is then given by

$$T_{12} = T_1 + T_2/G_1 \tag{3}$$

where $T_2$ and $G_1$ are both negative if the mixer has negative output resistance. Both $T_1$ and $G_1$ are properties of the mixer and are independent of the IF load impedance, but the IF noise temperature $T_2$ is dependent upon the mixer output impedance $Z_{out}$. For a given $Z_{out}$, we wish to find the lossless coupling network which minimizes $T_2$. For this purpose the IF noise temperature $T_2$ is most conveniently expressed in the form

$$T_2 = T_{min} + NT_0|Z_{out} - Z_{opt}|^2/(R_{out}R_{opt}) \tag{4}$$

where $T_{min}$, $N$, and $Z_{opt}$ are four noise parameters describing the IF amplifier and $T_0 = 290$ K. The noise parameter $N$ is equal to $g_n R_{opt}$, where $g_n$ is the noise conductance. This form is used because $T_{min}$ and $N$ are invariant to variation of a lossless coupling network which can be used to transform $Z_{opt}$ to whatever value minimizes $T_2$. By differentiating $T_2$ with respect to $R_{opt}$, we find the optimum positive values of $R_{opt}$ and the resulting minima of $T_2$ as follows:

$$R_{opt} = R_{out} \qquad T_2 = T_{min} \qquad \text{for } R_{out} > 0 \tag{5}$$

$$R_{opt} = -R_{out} \qquad T_2 = T_{neg} = T_{min} - 4NT_0 \qquad \text{for } R_{out} < 0. \tag{6}$$

For all cases $X_{opt} = X_{out}$. It can be shown as a property of noise parameters [6] that $T_{min} < 4NT_0$ and thus that the optimum $T_2$ for negative $R_{out}$, which we will call $T_{neg}$, is always negative. Furthermore, since $T_{neg}$ is a maximum, $T_2$ is always negative for negative $R_{out}$. It is also interesting to note that the optimum value of $R_{opt}$ is a positive value equal to the absolute value of $R_{out}$, and that $T_{neg}$ is invariant to a lossless coupling network since both $T_{min}$ and $N$ are invariant. Thus, $T_{neg}$ is a property of the active device or nonreciprocal elements in the amplifier. These results are illustrated in Fig. 2.

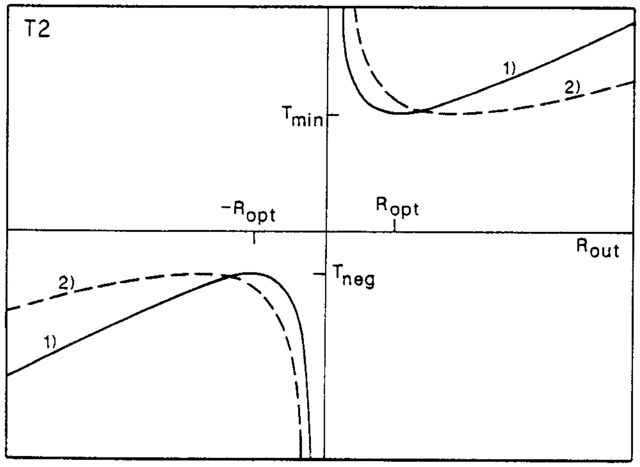


Fig. 2. The general form of the noise temperature of an amplifier as a function of generator resistance $R_{out}$ is shown above. Adjustment of the impedance transformation of the amplifier input network can change the location of $R_{opt}$, as from curve 1) to 2), but not the value of $T_{min}$. For negative generator resistance, the noise temperature is negative and has a maximum value of $T_{neg}$ at a generator resistance of $-R_{opt}$. The temperature, $T_{neg}$, is related to the noise wave coming out of the *input* port of the amplifier and may have absolute value higher or lower than $T_{min}$ dependent upon noise properties of nonreciprocal or active elements in the amplifier

A further physical interpretation can be given to $T_{neg}$ by examination of the noise wave model of a noisy amplifier [11]. This model represents the noise in the amplifier by two correlated noise waves having temperatures, $T_a$ and $T_b$, coupled in and out of the amplifier input terminals, respectively; close inspection reveals that $T_b = -T_{neg}$ if $Z_{opt} = Z_0$. (Note that in [11] $T_d = 4NT_0)/(1-|\Gamma_0|^2)$.) The total noise coming out of the amplifier input port is not just $T_b$, but also contains a contribution of $T_a$ reflected from the amplifier input terminals. However, in the case of an amplifier with an ideal input isolator with termination temperature $T_i$, then $T_b = -T_{neg} = T_i$. In this case we see that $T_{neg}$ is independent of $T_{min}$. A very noisy amplifier with a cold input isolator could produce very little noise when driven by a negative resistance with magnitude close to the isolator characteristic impedance. This case can be understood as a negative-resistance amplifier which precedes the amplifier. Another viewpoint is to consider the isolator as part of the mixer with impedances chosen for high gain. In practice, the only problem with this case is that the isolator forces the input resistance $R_{in}$ to be equal to $R_{opt}$, and as $R_{out}$ becomes close to $-R_{opt}$ for low noise, it will also become close to $-R_{in}$ and produce high and unstable gain. A better approach may be to design an amplifier with low $R_{in}$ for stable gain and low $T_b = -T_{neg}$ and $R_{opt} = -R_{out}$ (instead of low $T_{min}$ and $R_{in} = R_{opt}$, as is normal practice).

Some examples of measured values of noise parameters at cryogenic temperatures are given in Table I. The MGF1412 values are for a complete three-stage amplifier with an input coupling network, while the FHR01FH values refer to the transistor alone. The low value of $-T_{neg}$ at 8.4 GHz for the HEMT is encouraging.

TABLE I

| Transistor | Mitsubishi MGF1412 GASFET | Fujitsu FHR01FH HEMT |
|---|---|---|
| Reference | [12] | [13] |
| Temperature | 15 K | 12.5 K |
| Frequency | 1.6 GHz | 8.4 GHz |
| $T_{min}$ | 7.4 K | 10.3 K |
| $4NT_0$ | 16.0 K | 13.9 K |
| $-T_{neg}$ | 8.6 K | 3.6 K |
| $R_{opt}$ | 50 | 4.6 |
| $X_{opt}$ | 0 | 17.0 |

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# Plotting Vector Fields with a Personal Computer


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***Abstract*—A combination of a personal computer and an $x-y$ plotter is utilized for generating high-quality graphical displays of vector fields. The input data file must specify values of the vector field at a number of equidistant points. Starting points of field lines are determined so that the partial fluxes between any two adjacent field lines are made equal to each other. The construction of the field lines proceeds in a two-step method, utilizing the interpolated values of the field data.**



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## I. Introduction

A graphical display of electric and magnetic vector fields is indispensable for understanding the operation of many microwave devices. This short paper is concerned with the display of vector fields only. Computer programs for plotting the equicontours of scalar fields are already commercially available [1].

Examples of vector-field displays, generated by computer, can be found in recent references [2]–[4]. These references demonstrate that the smooth plots of vector fields can be obtained from discretized numerical data. The same references also demonstrate that a considerable initial programming effort is necessary and that, as a rule, a mainframe computer is required for implementation.

A systematic approach to graphical displaying of vector fields is possible if the program for computation of the field values is separated from the program which generates the field plots. The separation can be accomplished by requiring the field evaluation program to create a file of the vector-field values at a set of equidistant points, and then by devising a program which draws the smooth lines between these points. Such a general-purpose plotting program, reported in [5] and [6], was written for a mainframe computer and, in addition, it required the use of a large-size, general-purpose graphics software. Gradually, it was realized that the same task can be accomplished with considerably more modest equipment, such as a combination of a personal computer and an $x-y$ plotter. Although ordinary monitor displays on personal computers provide a very limited resolution (200 pixels vertically), the actual plotting accuracy is independent of the screen resolution. Even the modest $x-y$ plotters provide an excellent resolution of 0.025 mm [7]. Therefore, the personal computer can generate plots of a quality equal to those generated by mainframe computers and related hardware.

An algorithm for plotting the field lines and a procedure for selecting the starting points will be described, so that the density of field lines becomes a measure of the intensity of the vector field. The procedure is well suited for implementation on personal computers.

## II. Construction of Field Lines

The field lines are plotted in a plane, described in terms of Cartesian coordinates $x$ and $y$. In the case of an electric field, the components are denoted $E_x$ and $E_y$:

$$E(x,y)=\hat{x}E_x(x,y)+\hat{y}E_y(x,y). \tag{1}$$

For time-harmonic fields, $E$ is complex, and the instantaneous (real) value of the field must be computed by taking the real part of the expression $E(x,y)\exp(j\omega t)$. Each component of the vector field is assumed to be a quasi-linear function of position, for instance,

$$E_x(x,y)=C_1x+C_2y+C_3+C_4xy. \tag{2}$$

The four coefficients $C_1$ to $C_4$ can be computed from the known field values at the four corner points of an individual cell in a grid of equidistant data points.

The inclination angle $\theta$ of the field vector $E$ is specified by

$$\tan\theta=E_y/E_x. \tag{3}$$

Vector fields are represented by continuous field lines, also called lines of force [8, p. 161]. The line of force is tangential to the direction of the vector to be plotted. The differential equation